

TOWARDS BRANE-ANTIBRANE INFLATION IN TYPE II_A : THE HOLOGRAPHIC MQCD MODEL

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ABSTRACT

We describe type II_A cosmological brane inflation scenarios based on the holographic MQCD model of Aharony et al [1]. The scenarios can be related via T-duality to the type II_B KKLM model [2]. They describe a probe brane configuration of p D4 branes stretching between an $NS5$ and $NS5'$ branes in the holographic background of large N D4 branes. The resulting cosmological models have a Wick-rotated D4-brane metric, with transverse dimensions compactified, and a spiralling brane with flux p . In one model, the background has a small nonextremality, and the inflaton is provided by the position of a “sliding” D4-brane, and in the other, the background is supersymmetric, but with a sliding anti-D4-brane. We obtain good and generic inflationary models, though several unknowns remain, in particular about subleading corrections. The usual caveat of volume stabilization generically spoiling slow-roll still applies.

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1 Introduction

The inflationary scenario is by far the most successful in describing cosmological observations. However, embedding it in string theory with sufficient generality has proven to be quite difficult. One possibility to achieve it is brane-antibrane inflation, but usually one has to arrange for fine-tuned initial condition to obtain good slow-roll inflation [3, 4]. Another issue is obtaining a de Sitter vacuum in string theory in a controllable way, since a supersymmetric vacuum will be Anti-de Sitter. In [5], such a scenario was proposed, and since then other examples have emerged. This idea was then used in [2] to construct a model of brane-antibrane inflation in type IIB that satisfies both slow-roll and generality (absence of fine-tuning), though it was found that generically imposing stabilization of the last modulus (volume) spoils slow-roll. In view of this, it is useful to construct other models of inflation, even if they suffer from one of the above mentioned problems.

The Kachru-Kalosh-Linde-Trivedi (KKLT) model [5] is a type IIB model obtained by compactifying the Klebanov-Strassler (KS) solution [6], i.e. cutting off the KS cigar, a deformed cone with a base $T^{1,1}$ (a 5 dimensional space, topologically $S^2 \times S^3$) and radial direction r , at a certain value of r and gluing a CY_3 . Because of the $\mathcal{N} = 1$ supersymmetry of the KS solution, the gluing procedure (compactification) occurs smoothly, without extra energy needed at the junction. It is assumed that the dilaton and all the complex structure moduli are stabilized, and the only modulus left is the volume modulus. The background solution is obtained from a large number N of D3-branes, with fluxes and other branes (D7-branes, Euclidean D3-branes) added in a supersymmetric configuration. An anti-D3-brane is added at the tip of the KS solution, breaking supersymmetry, and lifting the AdS minimum to a dS minimum.

Then, Kachru-Kalosh-Linde-Maldacena-McAllister-Trivedi (KKLMMT) [2] considered a modification of the KKLT model, where there is an extra D3-brane sliding towards the anti-D3-brane at r_{min} in the compactified KS geometry. One obtains a model of D-brane anti-D-brane inflation, where the inflaton is the separation between the brane and the antibrane, though it is found that generically requiring stabilization of the volume modulus spoils the nice features of the brane-anti-brane potential.

In the following we will search for a IIA counterpart to the IIB KKLMMT model, looking for inflation with the inflaton = position of a probe brane. The same caveat as for KKLMMT applies to our model, namely requiring stabilization of the volume modulus will spoil the flatness of the potential. In this paper we will not discuss how to deal with this problem.

In this present paper we analyze inflation scenarios based on type II_A models. The models are based on two versions of holographic MQCD that were proposed in [1]. These models consist of an uplift to M theory of a type II_A brane configuration depicted in figure (1). In both models a world-volume coordinate of the D4 branes x_6 is compactified. In one case, the extremal case, supersymmetry is unbroken and the geometry in the radial and x_6 direction is that of a cylinder (see figure (4)). In the other case, the near-extremal one, supersymmetry is slightly broken by a small nonextremality parameter, yielding a cigar-like geometry of figure (3) (in the limit of zero non-extremality, we recover the supersymmetric case).

In both cases the form of the “probe p D4 brane plus the NS and NS’ branes” take the form of a spiralling brane descending up to a certain radial value and then ascending back (see figures 3 and 4), but we will find that its effect on the inflaton potential is a small correction, so we can just use the supersymmetric or near-supersymmetric backgrounds of above without the spirals. Our first inflation scenario is based on the latter case where a sliding D4 brane is added and its location along the radial direction plays the role of the inflaton. In the second scenario the inflaton is the location of anti-D4 brane in the supersymmetric background.

We should mention that, despite the fact that for the supersymmetric case the background we use is related in the limit of a shrinking radius by T-duality to the KS background, the mechanism of generating the inflaton potential cannot be obtained directly from the KKLMMT one, and hence the scenario described here is a novel one.

The paper is organized as follows. In section 2 we will describe the set-up T-dual to KKLMMT, of a Wick-rotated near-extremal D4-brane with a spiralling 5-brane added, and the position of a moving D4-brane providing the inflaton. Adding the spiral makes a small correction to the potential. In section 3 we describe an alternative model, with an extremal D4-brane background and a moving anti-D4-brane. In section 4 we analyze the resulting cosmology of these models, and in section 5 we conclude.

2 A model based on a sliding D brane in a non-supersymmetric (cigar) background

2.1 The basic set-up

If we T-dualize the KS solution to type IIA on a coordinate x_6 , we obtain a solution made up of a large number N of D4-branes wrapping x_6 . In [1], the solution T-dual

to KS was described by a background of N D4-branes, lifted to M theory as M5-branes, in which one has other (probe) p D4-branes stretched between an NS5-brane and an NS5' brane.

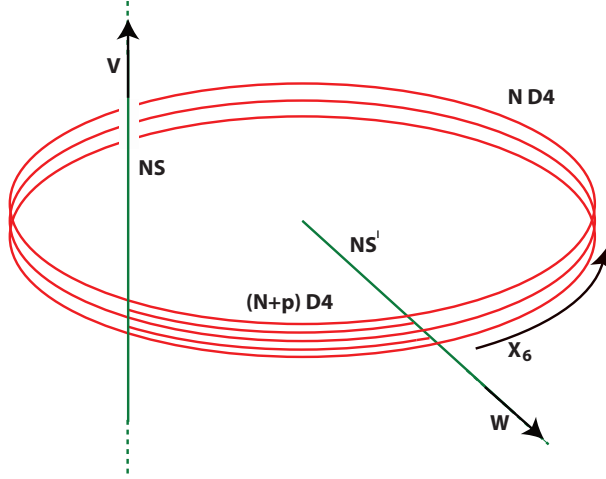


Figure 1: N D4-branes that wrap the x_6 circle, and p that are stretched between the NS and NS' -branes.

The p probe D4-branes and 2 NS5-branes lift to M theory as a single 5-brane, winding around x_6 . In the cylinder formed by x_6 and the transverse partial radial coordinate u , the resulting 5-brane spirals down to an u_{min} , and then goes back up. A particular limit of the background of N D4-branes with the spiralling brane embedded was shown to be equivalent to the KS construction for the T-dual type IIB.

The 5 coordinates transverse to the N D4-branes are described by 2 complex variables v and w , with the (partial) radial coordinate u , $u^2 = |v|^2 + |w|^2$ and another one x_7 , with the overall transverse coordinate r , $r^2 = u^2 + x_7^2$. Therefore the transverse space is sort of a semi-infinite cigar with radial coordinate r and base S^4 .

In the KKLT construction there is also an anti-D3 brane at the tip of the KS solution, breaking susy in a controllable way. In the model we study here, we choose to make the nonsusy perturbation a part of the background, by taking near-extremal D4-branes. More precisely, since we are interested in preserving the 3+1d Minkowski invariance, we take the double-Wick rotated solution, along t and x_6 , giving the background

$$\begin{aligned} ds^2 &= H_4^{-1/2}(r)(-dt^2 + d\vec{x}_3^2 + f(r)dx_6^2) + H_4^{1/2}\left(\frac{dr^2}{f} + r^2 d\Omega_4^2\right) \\ e^{2\phi} &= g_s^2 H_4^{-\frac{1}{2}} \end{aligned}$$

$$\begin{aligned}
C_4 &= \frac{1}{g_s} H_4^{-1} dt \wedge dx^1 \dots \wedge dx^4 \\
f &= 1 - \left(\frac{r_H}{r} \right)^3 \\
H_4 &= 1 + \alpha_4 \left(\frac{r_4}{r} \right)^3
\end{aligned} \tag{2.1}$$

This way of breaking susy was considered in [1], and it gives rise to a semi-infinite cigar geometry in r over S^4 , cut off at $r_{min} = r_H$, as well as a cigar geometry in r over x_6 (see figure (3)). We can then consider again (as in [5]) cutting off the cigar at a certain r_{max} , and gluing another space. The set-up thus obtained would correspond to KKLT (compactified KS with a susy breaking anti-D3 at the tip). But we want to consider inflation in the KKLMNT set-up, therefore we will add a sliding D4-brane along the cigar, whose position will be our inflaton.

Considering that the cigar including the T-duality coordinate x_6 , (x_6, r, S^4) is the space T-dual to the $(r, T^{1,1})$ cigar in KKLT, which is glued onto a CY_3 space M . Then (x_6, r, S^4) needs to be glued onto the CY_3 space W T-dual to M (note that there is no simple $U(1)$ in general, so T-duality is generalized). Strictly speaking, the T-duality picture holds for the supersymmetric case $f = 1$ (or $r_H = 0$), but it will be approximately valid in the near-extremal case $r_H \ll r_4$. Note that if we perturb a bit an exact T-duality symmetry by the introduction of a small non-extremality parameter, we still get an approximate symmetry. Also, the compactification by gluing of W is supersymmetry preserving, i.e. the gluing does not generate extra energy. In the near-extremal case, the gluing will not be perfect, so a small additional energy needs to be added to realize it, but we will argue later that we can neglect it.

We will consider the near-extremal case in the following, for the previous reasons, as well as a number of others that will become apparent as we analyze the model.

Also, for the T-duality to KS to work, we must add the spiralling brane. But we will see that we obtain a good inflationary model by just considering the background (2.1), and the effect of the spiralling brane can be constrained to be small. Therefore we will first analyze the case without the spiral, and then move to the spiral perturbation. The analysis of the effect of the spiral is nevertheless important, since the spiral guarantees the T-duality with KS, which is known to be well-defined due to its supersymmetry.

2.2 Near-extremal D4-brane

The action for a Dp-brane which includes the DBI term and the CS term takes the form

$$S_p = -T_p \int e^{-\phi} \sqrt{\det G_{ab}} + \mu_p \int C_{p+1} \tag{2.2}$$

and for a Dp-brane moving in the background of the double-Wick rotated non-extremal Dp-branes one obtains

$$S = -\frac{T_p}{g_s} \int H_p^{-1}(r) \left[\sqrt{f_p + \frac{H_p(r)}{f_p} g^{\mu\nu} \partial_\mu r \partial_\nu r} - 1 \right] \quad (2.3)$$

where H_p and f_p are the analogs of H_4 and f given in (2.1). The corresponding potential for D4-branes ($p = 4$) acting on a D4-brane sliding in the background (2.1) is

$$V_4(r) = +\frac{T_4 R}{g_s} H^{-1}(r) [\sqrt{f(r)} - 1] = +\frac{T_4 R}{g_s} \frac{1}{1 + \alpha_4 (\frac{r_4}{r})^3} \left[\sqrt{1 - \frac{r_H^3}{r^3}} - 1 \right] < 0 \quad (2.4)$$

Here R is the radius of the compact x_6 without the metric factors, r is the radial position of the sliding brane and

$$\begin{aligned} (r_4)^3 &= \pi g_s N \alpha'^{3/2} \\ \alpha_4 &= \sqrt{1 + \left(\frac{r_H^3}{2r_4^3}\right)^2} - \frac{r_H^3}{2r_4^3} \end{aligned} \quad (2.5)$$

so that in the near-extremal case $r_H \ll r_4$, $\alpha_4 \simeq 1$. The potential is drawn in figure 2.

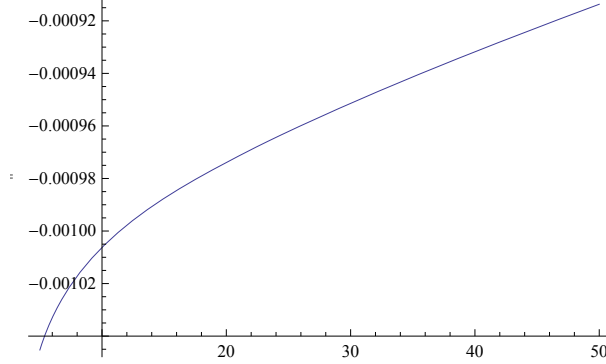


Figure 2: The potential $V(r)$ in the plateau region, as a function of r for $\beta = 500$. The value of $V(r)$ on the plateau is $1/2\beta = 0.001$, and at infinity, $V(r)$ goes to zero.

To analyze this potential, we compute its derivative,

$$\frac{g_s V'(r)}{T_4 R} = \frac{3}{2r^4 (1 + \alpha_4 (\frac{r_4}{r})^3) \sqrt{1 - \frac{r_H^3}{r^3}}} \left[\frac{2\alpha_4 r_4^3}{1 + \alpha_4 \frac{r_4^3}{r^3}} \left(1 - \frac{r_H^3}{r^3} - \sqrt{1 - \frac{r_H^3}{r^3}} \right) + r_H^3 \right] \quad (2.6)$$

Denoting $r_H^3/r^3 \equiv x$, $\alpha_4 r_4^3/r_H^3 \equiv \beta$, we can check that $V'(r) > 0$, since from $V'(r) = 0$ we get the equation $1 + \beta x + 2\beta(1 - x - \sqrt{1 - x}) = 0$, implying $\beta^2 x^2 - 2\beta x + (1 + 2\beta) = 0$, which has a negative discriminant, therefore no solution. That means that the potential increases monotonically from $r = r_H$, where it takes the value

$$V(r_H) = -\frac{T_4 R}{g_s} \frac{1}{1 + \alpha_4 \left(\frac{r_4}{r_H}\right)^3} \simeq -\frac{T_4 R}{g_s} \left(\frac{r_H}{r_4}\right)^3, \quad (2.7)$$

to infinity, where it gives zero. Near $r = r_H$, the potential becomes very steep, with $V'(r = r_H) = \infty$, but the value of the potential stays finite.

Far from the horizon r_H , at $r/r_H \gg 1$, we get

$$V(r) \simeq -\frac{T_4 R}{2g_s} \left(\frac{r_H}{r}\right)^3 \frac{1}{1 + \alpha_4 \left(\frac{r_4}{r}\right)^3} \quad (2.8)$$

We observe then that even though $V'(r)$ stays always positive, it does in fact stay very close to zero over a large region, if $r_4 \gg r_H$, i.e. if $\beta \gg 1$, since then the potential is approximately

$$V(r) \simeq -\frac{T_4 R}{2g_s \alpha_4} \left(\frac{r_H}{r_4}\right)^3 \left[1 + \frac{1}{4} \frac{r_H^3}{r^3} - \frac{r^3}{\alpha_4 r_4^3}\right] \quad (2.9)$$

so is approximately constant over the large region $r_H \ll r \ll r_4$, with $V(r) \simeq V(r_H)/2$.

Note that the potential is negative, but it should really be positive, due to the nonsupersymmetric deformation. The answer to this puzzle is that the potential we derived is not yet complete. There is also a vacuum energy component, due to the nonsupersymmetric nature of the background. We are interested in the energy from the point of view of the effective 4d theory in flat space, which means that this energy needs to be positive since the supersymmetric theory would have zero energy.¹ This value will depend on the volume of compactification, but if this volume is large enough the dependence will become negligible, and we will have the vacuum energy of the uncompactified theory, so we will focus on this in the following.

To compute this energy, one would need to regularize an integral, corresponding to the gravitational action, over an infinite volume. Since this regularized gravitational action could be calculated in Euclidean space, and then Wick-rotated, the fact that our solution is doubly-Wick rotated with respect to the near-extremal solution (we

¹Of course, the total (Casimir) energy of a gravitational space can be negative even in a non-supersymmetric set-up. However, we are interested in the energy from the point of view of the effective theory in flat 4 dimensions, and we know that in that case, supersymmetry requires zero energy. This is exactly the same situation as was encountered in [7], so a similar reasoning applies.

exchanged t with x_6) should not matter, and we should get the same result. Luckily, a simpler procedure for calculating this energy in the near-extremal case was devised in [7] in order to compute the vacuum energy from the point of view of the 4d theory in flat space, that guarantees a finite result. The one difference is that in that case, because one was using AdS/CFT, the theory was defined in the IR of the dual metric (because of the UV/IR relation between gravity and field theory), i.e. both g_s and the size of the space wrapped by the branes was defined at small r , whereas in our case g_s and R are defined at large r , since we have just an effective field theory picture.

A non-extremal solution is obtained by adding to the extremal solution (representing a set of D-branes), an extra mass δM without charge. That δM is equivalent to adding $\delta N = \delta M/2$ D-branes and δN anti-D-branes, since a D-brane and an anti-D-brane have mass, but no charge. Therefore we have

$$\frac{r_H^3}{r_4^3} = \frac{\delta M}{M} = \frac{2\delta N}{N} \quad (2.10)$$

The vacuum energy is the tensional energy of the $2\delta N$ branes wrapping the x_6 circle of radius R at coupling g_s , in the same way as in [7]. The result is then

$$E_0 = \frac{T_4 R}{g_s} 2\delta N = \frac{T_4 R N}{g_s} \frac{r_H^3}{r_4^3} \quad (2.11)$$

The total potential is then the sum of (2.4) and (2.11). Note that $E_0 = NV(r_H)$, so the resulting potential is very flat simply due to the large factor of N in the constant part. Also note that we will have an a posteriori check on this calculation in the second model we will analyze, in eq. (3.2).

Since the background is slightly nonsupersymmetric, in order to compactify by gluing to a CY space we would need to slightly modify the gluing region, which will create an additional energy, localized near the gluing region. But such an energy will be proportional to $\delta N/N$, as it should vanish in the supersymmetric case, yet it cannot be proportional to N also, since the gluing region is not drastically modified by an increase in N (at fixed $\delta N/N$, the space in the absence of the cut-off r_{max} would just scale up, but at fixed cut-off r_{max} , that would be equivalent to scaling the cut-off instead). But at large r_{max} , the space is approximately supersymmetric, and if the energy was proportional to N , by the previous argument, it would mean that the energy could be made infinite by just scaling the cut-off, which is clearly absurd. Hence the energy is not proportional to N . This means that such a contribution will be much smaller than (2.11). It could be comparable to (2.9), but that does not matter, since it is a constant contribution, so all that matters is the relation to (2.11). We can thus safely ignore it.

2.3 Adding the spiralling brane

We now calculate the contribution of the spiralling brane to the potential.

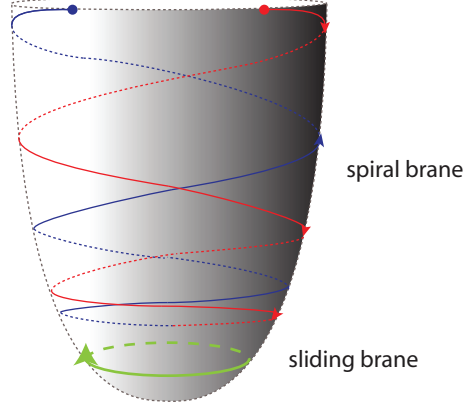


Figure 3: The spiralling profile over the cigar background. We use a red line for the “downward” spiralling brane, a blue one for the climbing one and a green one for the sliding brane.

Since the type IIA configuration is quite complicated-looking, it is easier to do the calculation in M-theory. We can use the same idea used in [2] to calculate the potential between the D3 and the anti-D3 in the KS geometry. It was noted that the harmonic function $H(r)$ is harmonic in the transverse space of the background metric, $dr^2 + r^2 \tilde{g}_{ab} dy^a dy^b$, and as a result one can calculate the perturbation of the harmonic function due to the supersymmetric D-brane at a position r_1 in the transverse space. We can use then the probe approximation for the anti-D-brane, and compute the interaction potential via the modification of its DBI action induced by the perturbation of $H(r)$. We can reverse the logic and consider the perturbation to the background given by the anti-D-brane, and calculate its effect on the D-brane. In [8] it was checked that this gives the same result in a rather large class of situations. The anti-D-brane would turn the background into a near-extremal one.

In our IIA case, the latter situation corresponds to having the near-extremal background together with the spiral, and calculate its effect on the sliding D4-brane. But we can now switch the points of view yet again, and consider instead the effect of the sliding brane, which would be supersymmetric for $f = 1$, on the background, and calculate the modification of the action of the spiralling brane. This last switching of points of view is on an even surer footing, since the sliding and spiralling branes are of the same type (branes, not antibranes).

Of course, for this approximation to hold, we should be able to consider the non-extremality as being the effect of only a few anti-D-branes, i.e. to be in the

near-extremal case. We should also note that we will treat the modification of the near-extremal background due to the probe brane as a modification of the harmonic function H (and possibly r_H) only. This should be correct in the near extremal case only. If the probe brane would be situated at $r = 0$ like the rest, only H would be modified, independent of the non-extremality. For our probe brane at $r > r_H$, there can be other changes besides the change in H (and possibly r_H), but they can only be of higher order in $1/N$, since the modifications of the other parameters have to be proportional to the near-extremality parameter $\delta N/N$, whereas the supersymmetric effect (modification of H only) is already proportional to $1/N$, so in total we must have at least $\delta N/N^2$. We will therefore neglect such modifications in the following. Note that this argument relies on the fact that the probe is a brane, which would be in a supersymmetric configuration in the absence of the small nonextremality of the background, and thus it will only modify the parameters of the background (H and r_H), but not its form.

Since, as we said, it will be easier to work in M theory, where the spiral is a simple M5-brane, we consider the M theory uplift of the background,

$$\begin{aligned}
ds^2 &= H^{-1/3}[-dt^2 + dx_i^2 + f(r)dx_6^2 + dx_{11}^2] + H^{2/3}[(f(r)^{-1} - 1)dr^2 \\
&\quad + |dv|^2 + |dw|^2 + dx_7^2] \\
C_6 &= H^{-1}d^4x \wedge dx_6 \wedge dx_{11} \\
r^2 &= |v|^2 + |w|^2 + x_7^2
\end{aligned} \tag{2.12}$$

We have a M5-brane situated at $x_7 = 0$, so that $r = u$, with

$$\begin{aligned}
v &= u(x_6)e^{i\phi(x_{11})}\cos\alpha(x_6) \\
w &= u(x_6)e^{-i\phi(x_{11})}\sin\alpha(x_6)
\end{aligned} \tag{2.13}$$

and winding around x_6 many times, spiralling down in u to an u_{min} , and then back up.

The fact that in type IIA this describes p D4-branes is realized via the fact that the M5-brane wraps p times around x_{11} , so $\phi(x_{11}) = x_{11}/\lambda_p$ (the equation of motion of ϕ is $\ddot{\phi} \equiv \partial_{x_{11}}^2 \phi = 0$), where

$$\lambda_p = pg_s l_s = pR_{11} \tag{2.14}$$

The M5-brane action is [9, 10]

$$\begin{aligned}
S &= -T_5 \int d^6x \left[\sqrt{-\det(g_{mn} + \tilde{H}_{mn})} - \sqrt{-g} \frac{1}{4\partial_r a \partial^r a} \partial_t a H^{*lmn} H_{mnp} \partial^p a \right] \\
&\quad + \int [C^{(6)} + \frac{1}{2}F \wedge C^{(3)}]
\end{aligned}$$

$$\begin{aligned}
H &= F - C^{(3)} \\
\tilde{H}_{mn} &= \frac{1}{\sqrt{-(\partial a)^2}} H_{mnl}^* \partial^l a \\
H^{*mnl} &= \frac{1}{3! \sqrt{-g}} \epsilon^{mnlpqr} H_{pqr} \\
*dC^{(3)} &= dC^{(6)} + \frac{1}{2} C^{(3)} \wedge dC^{(3)}
\end{aligned} \tag{2.15}$$

where a is an auxiliary scalar field, needed to avoid explicit breaking of Lorentz invariance, and whose VEV gives for instance $\partial_l a = \delta_{l5}$, but on our background it reduces to only

$$S = -T_5 \int d^6 x \left[\sqrt{-\det g_{\mu\nu}^{ind}} - C^{(6)} \right] \tag{2.16}$$

Since we want to vary the harmonic function, we calculate the action of the M5-brane in the above background as a function of H , without substituting its value, obtaining

$$-\frac{g_s \mathcal{L}}{T_4} = H^{-1} \sqrt{1 + H(u\dot{\phi})^2} \sqrt{f + \frac{H}{f} [(u\alpha')^2 + u'^2]} - H^{-1} \tag{2.17}$$

where $T_4 = T_5 2\pi R_{11}$. Here prime refers to $\partial/\partial x_6$ and dot to $\partial/\partial x_{11}$. The two integrals of motion corresponding to translational invariance in x_6 and α are then

$$\begin{aligned}
E &= p_u u' + p_\alpha \alpha' - \mathcal{L} = H^{-1} - \frac{H^{-1} \sqrt{1 + H(u\dot{\phi})^2} f}{\sqrt{f + \frac{H}{f} [(u\alpha')^2 + u'^2]}} \\
J &= p_\alpha = \frac{\sqrt{1 + H(u\dot{\phi})^2}}{\sqrt{f + \frac{H}{f} [(u\alpha')^2 + u'^2]}} \frac{u^2 \alpha'}{f}
\end{aligned} \tag{2.18}$$

We can then eliminate u' and α' in favor of E and J , obtaining

$$\begin{aligned}
u\alpha' &= \frac{f^2}{1 - HE} \frac{J}{u} \frac{1}{\lambda_p} \\
u' &= \frac{f}{1 - HE} \sqrt{f \left(H^{-1} + u^2 - \frac{f J^2}{u^2} \right) - H^{-1} (1 - HE)^2}
\end{aligned} \tag{2.19}$$

where we fixed $\dot{\phi} = 1$ by a rescaling,

$$u = \lambda_p \bar{u} \quad u_4 = \lambda_p \bar{u}_4 \quad x_6 = \lambda_p \bar{x}_6 \tag{2.20}$$

and dropped the bars here and in the following. Substituting in the action, we obtain

$$-\frac{g_s \mathcal{L}}{T_4} = \frac{H^{-2} (1 + H u^2) f}{H^{-1} - E} - H^{-1} \tag{2.21}$$

The potential as a function of the position u_0 of the sliding brane is

$$V(u_0) = -2\lambda_p \int_0^\infty dx_6 \mathcal{L}(u_0, u(x_6)) \quad (2.22)$$

since the brane wraps around x_6 starting at infinity in u , down to u_{min} (corresponding to $x_6 = 0$) and then back up to infinity. Substituting the Lagrangian, we obtain

$$V(r_0) = \frac{2T_4\lambda_p}{g_s} \int_0^\infty dx_6 \left[\frac{H^{-2}f(1+Hu^2)}{H^{-1}-E} - H^{-1} \right](x_6) \quad (2.23)$$

If we put $f = 1$ and $E = 0$ we return to the supersymmetric case, and then we have [1]

$$\begin{aligned} u' &= \frac{1}{\lambda_p} \sqrt{u^2 - \frac{J^2\lambda_p^2}{u^2}}; \quad J\lambda_p \equiv 2\xi^2 \\ u^2 &= 2\xi^2 \cosh \frac{2x_6}{\lambda_p} = J\lambda_p \cosh \frac{2x_6}{\lambda_p} \end{aligned} \quad (2.24)$$

(here and in (2.25) u and x_6 are unbarred quantities), in which case by substituting we obtain an infinite constant,

$$V = +2T_4 \int_0^\infty dx_6 u^2 = +2T_4 \xi^2 \lambda_p \sinh \left(\frac{2x_6}{\lambda_p} \right) \Big|_0^\infty \quad (2.25)$$

Since we have no interaction in the supersymmetric case, this is just the rest mass of the spiralling brane, and is divergent since the spiralling brane is infinite in extent in x_6 . In a physical case, we must make it finite by regularizing: we integrate $\int_0^{2\pi Rk} dx_6$, or correspondingly $\int_{u_{min}}^\Lambda du$, obtaining

$$V = +2T_4 \xi^2 \lambda_p \sinh \frac{4\pi Rk}{\lambda_p} \simeq +\lambda_p T_4 \Lambda^2 \quad (2.26)$$

where the last equality is only valid at large Λ .

In order to calculate the interaction potential between the spiral and the sliding brane, we calculate the change in V due to the variation in the harmonic function. There is also a variation of r_H of order $1/N$, but we will neglect it at this time, and come back to it at the end of this subsection.

The harmonic function corresponds to N branes at $r = 0$, and the sliding brane adds another one at r_0 , therefore

$$H \rightarrow H + \delta H; \quad \frac{\delta H}{H} \simeq \frac{r^3}{N(r-r_0)^3} \quad (2.27)$$

The interaction potential is then

$$\delta V(r_0) = +\frac{2T_4\lambda_p}{g_s} \int_0^\infty dx_6 \frac{r^3}{N(r-r_0)^3} H^{-1} \left\{ 1 - \frac{H^{-1}f}{(H^{-1}-E)^2} \left[H^{-1} - E(2+Hu^2) \right] \right\} \quad (2.28)$$

We can check that in the supersymmetric case $f = 1$, $E = 0$, the interaction potential vanishes.

Substituting u' from (2.19) in (2.28), we finally get for the interaction potential between the sliding brane and the spiral

$$\begin{aligned} \delta V_{E,J}(r_0, \Lambda) &= +\frac{2T_4\lambda_p}{g_s N} \int_{u_{min}(E,J)}^\Lambda \frac{du(1-HE)}{f\sqrt{f(H^{-1}+u^2-\frac{fJ^2}{u^2})-H^{-1}(1-HE)^2}} \times \\ &\times \frac{r^3}{(r-r_0)^3} H^{-1} \left\{ 1 - \frac{H^{-1}f}{(H^{-1}-E)^2} \left[H^{-1} - E(2+Hu^2) \right] \right\} \end{aligned} \quad (2.29)$$

Here u_{min} is the turning point for the spiral, which depends on E, J by solving the equation $u'(u) = 0$, i.e.

$$\begin{aligned} &\left[f(H^{-1}+u^2-\frac{fJ^2}{u^2})-H^{-1}(1-HE)^2 \right]_{u=u_{min}} = 0 \Rightarrow \\ &\left(1 - \frac{u_H^3}{u_{min}^3} \right) \left[\frac{1}{1+\alpha_4 u_4^3/u_{min}^3} + u_{min}^2 - \left(1 - \frac{u_H^3}{u_{min}^3} \right) \frac{J^2}{u_{min}^2} \right] \\ &= \frac{1}{1+\alpha_4 u_4^3/u_{min}^3} \left(1 - \frac{E}{1+\alpha_4 u_4^3/u_{min}^3} \right)^2 \end{aligned} \quad (2.30)$$

The potential thus depends on r_0 and Λ as variables, and E and J as constants parametrizing the solution. Since Λ is the value of u where we glue to the CY_3 , Λ is related to the volume variable.

In the case that Λ is sufficiently large so that $H(\Lambda) \simeq 1$, $f(\Lambda) \simeq 1$, the interaction potential contains a divergence,

$$\delta V_{E,J}(r_0, \Lambda)_{div} \sim +\frac{2T_4\lambda_p E}{g_s N(E-1)^2} \left[\frac{\Lambda^2}{2} + 3r_0\Lambda + \dots \right] \quad (2.31)$$

We are interested in the near-extremal case, which means that we can calculate as a perturbation around the susy case $E = 0$, $f = 1$. For small E and $f - 1$, we get for the potential (2.29)

$$\delta V_{E,J}(r_0, \Lambda) \simeq +\frac{2T_4\lambda_p}{g_s N} \int_{\sqrt{J}}^\Lambda \frac{u^3 du}{(u-u_0)^3} \frac{H^{-1}}{\sqrt{u^2-J^2/u^2}} \left[-(f-1) + EH^2 u^2 \right] \quad (2.32)$$

where we have used the fact that in the susy case (and thus also for the near-extremal case in this order of approximation) $u_{min} = \sqrt{J}$.

Again if we have a sufficiently large Λ such that $H(\Lambda) \simeq 1$, $f(\Lambda) \simeq 1$, there is the same divergence

$$\delta V_{E,J}(r_0, \Lambda)_{div} \sim + \frac{2T_4 \lambda_p E}{g_s N} \left[\frac{\Lambda^2}{2} + 3u_0 \Lambda + \dots \right] \quad (2.33)$$

The region of interest for r_0 is $r_H \ll r_0 \ll r_4$, and there $H(r_0) \simeq \alpha_4 r_4^3 / r_0^3$ and $f(r_0) \simeq 1$, but the integral in r is over a larger region, where the same does not apply. Therefore we would need to evaluate the integral numerically to get a result.

However, if the region of interest is such that $H(\Lambda) \gg 1$, we can approximate $H(u_0) \simeq \alpha_4 r_4^3 / u_0^3$, giving

$$\delta V_{E,J}(r_0, \Lambda) \simeq + \frac{2T_4 \lambda_p}{g_s N \alpha_4 r_4^3} \int_{\sqrt{J}}^{\Lambda} \frac{u^3 du}{(u - u_0)^3} \frac{1}{\sqrt{u^2 - J^2/u^2}} \left[\frac{E \alpha_4^2 r_4^6}{u} + r_H^3 \right] \quad (2.34)$$

(note that, since we took $H(\Lambda) \gg 1$ to obtain the above, $\alpha_4^2 r_4^6 \gg u^6$, so the first term is $\gg Eu^5$, i.e. dominates over the constant r_H^3 term at large u , so the leading behavior of the potential, giving the Λ behavior, is obtained from integrating the first term) and now we see that the contribution of large u goes like $1/\Lambda$, i.e. not only it does not diverges for $\Lambda \rightarrow \infty$, but it actually vanishes.

More importantly, we note that the result is of order $\sim g_s p / N$ with respect to (2.9), so we can consider it a small correction, and neglect it in the analysis of cosmology.

We now return to the issue of the possible variation of r_H of order $1/N$. Let's write generically $\delta r_H / r_H = \beta / N$. We can again vary the potential in (2.23) and obtain

$$\delta V \simeq - \frac{3\beta}{N} \frac{2T_4 \lambda_p}{g_s} \int_0^\infty dx_6 \frac{r_H^3}{r^3} \frac{u^2 + H^{-1}}{1 - HE} \quad (2.35)$$

Again substituting u' from (2.19) and taking a small E and $f - 1$, we obtain

$$\delta V \simeq - \frac{3\beta}{N} \frac{2T_4 \lambda_p}{g_s} u_H^3 \int_{\sqrt{J}}^{\Lambda} \frac{du}{u^3} \frac{u^2 + H^{-1}}{\sqrt{u^2 - J^2/u^2}} \quad (2.36)$$

We can now check that the integral has a large Λ dependence of $1/\Lambda$, and that it is finite at \sqrt{J} . Therefore the perturbation of the potential due to the variation in r_H is of the order of $(g_s p)^3 (g_s N)^{2/3} / N$ with respect to (2.9), therefore again sub-leading, and will be neglected in the analysis of the application to cosmology.

Finally, it is also clear that we can neglect also the modifications due to changes in other quantities besides the harmonic function H and r_H , since as we said those were expected to be even smaller than (2.34).

3 A model based on a sliding anti-brane in a supersymmetric (cylinder) background

In the previous section we have considered the case of a probe brane falling in the background of a doubly Wick rotated nonextremal D4-brane solution, which itself could be thought of as being made up of $N + \delta N$ D4-branes and δN anti-D4-branes (with a small number δN since we consider the near-extremal case $r_H \ll r_4$), with a spiral brane probe added on the background.

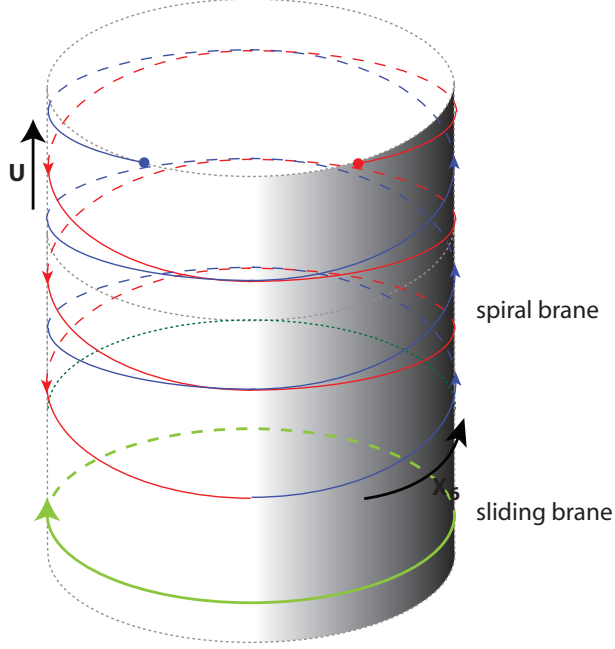


Figure 4: For compact x_6 , the curved $NS5$ -brane spirals down the (x_6, u) cylinder and then climbs back up. The downward (upward) part of the spiral is colored red (blue).

We consider now the case of a supersymmetric background of N D4-branes and a spiral brane added with a moving anti-brane probe giving the inflaton potential (see figure (4)). If we were able to exactly describe the case of $\delta N = 1$, we could argue as in [8], where it was argued that exchanging the brane with the antibrane as probe and part of the background respectively, would in most cases give the same result. In our case, it is less clear, so we will treat here separately this case. But the upshot is that at $\delta N = 1$ we could switch the role of the probe brane and $\delta N = 1$ antibrane in the background and get this second model. A better way of saying this would be that at $\delta N = 1$ we cannot think of the antibrane as part of the background anymore, but instead we should think of it as a probe in a supersymmetric background.

The compactification of the solution happens in exactly the same way as in the previous sections, since the supersymmetric case is T-dual to the KS background, as explained in [1]. The only difference is that since now we have a supersymmetric case, there is no problem anymore in matching the cylinder with the spiralling brane to the half of CY space. This will not introduce extra energy at the joining point. On the low r side, the only difference between this case and the previous one is that the space terminates at $r = 0$ instead of $r = r_H$.

The Einstein frame metric is

$$\begin{aligned} ds_E^2 &= H^{1/2} H^{-1/8} [dr^2 + r^2 d\Omega_4^2 + H dx_6^2] + \dots \\ &\sim r^{-9/8} [dr^2 + r^2 d\Omega_4^2 + r^3 dx_6^2] + \dots \\ &\sim dy^2 + y^2 d\Omega_4^2 + y^{30/7} dx_6^2 + \dots \end{aligned} \quad (3.1)$$

so the space terminates at $y = 16/7 r^{7/16} = 0$. Of course, the solution above is singular at $r = 0$, so there will be corrections to the geometry, but we will still have a space that terminates, so the extra dimensions are truly compactified by this construction.

The calculation of the potential is the same as before, just putting $r_H = 0$ and changing the sign of the CS term because we work with an anti-brane probe, i.e. changing the -1 to $+1$ in (2.4), obtaining

$$V_4(r) = \frac{2T_4 R}{g_s} \frac{1}{1 + \frac{r_4^3}{r^3}} \quad (3.2)$$

Now, since in the absence of the antibrane probe we have a supersymmetric model, we don't have any constant term to add. However, it is satisfying to observe that the constant term in the above potential, $V_4(\infty)$, equals the constant term we assumed in the previous section for $\delta N = 1$, as it should, since we argued that the two models are related just by the exchange of a $\delta N = 1$ brane part of the background and a brane probe.

The potential now varies between $r = 0$ where it is $V(0) = 0$, with the derivative giving

$$V'_4(r) = \frac{6T_4 R}{g_s r} \frac{r_4^3/r^3}{1 + r_4^3/r^3} \quad (3.3)$$

which goes to infinity at $r = 0$, and at infinity the potential flattens out to $V(\infty) = 2T_4 R/g_s$.

We next turn to the computation of the correction to the potential due to the spiralling brane. We would like to use the same logic as was used above for the

sliding brane on the non-supersymmetric cigar background, namely, instead of computing the impact of the spiral on the sliding anti-brane we will aim to compute the reverse, namely the impact of the anti-brane on the spiral, and moreover we do the computation using the effect of the probe on the background.

Had it been a sliding brane rather than an anti-brane the contribution to the potential of the spiralling brane would have been vanishing as we have seen in (2.28) when we take $f = 1$ and $E = 0$. This would follow from a cancelation between the DBI and CS contributions to the potential. Now for the potential acting on a sliding anti-brane the contribution of the DBI term is the same on the sliding brane but that of the CS term has an opposite sign. Thus altogether the contribution of the spiralling brane to the potential acting on the sliding anti-brane will be twice the contribution of the DBI to the potential acting on a sliding brane, namely

$$\begin{aligned}\delta V(r_0) &= \frac{4T_4}{g_s} \int_0^\infty dx_6 \frac{r^3}{N(r-r_0)^3} H^{-1} \\ &= \frac{2T_4\lambda_p}{g_s N} \int_{\sqrt{J}}^\Lambda \frac{u^3 du}{(u-u_0)^3} \frac{H^{-1}}{\sqrt{u^2 - J^2/u^2}} \\ &\simeq \frac{4T_4\lambda_p}{g_s N \alpha_4 r_4^3} \int_{\sqrt{J}}^\Lambda \frac{u^6 du}{(u-u_0)^3} \frac{1}{\sqrt{u^2 - J^2/u^2}}\end{aligned}\tag{3.4}$$

where in the last equality we took the near horizon harmonic function $H \simeq \alpha_4 \left(\frac{r_4}{u}\right)^3$. We see that in this case, we get a very strong Λ dependence (volume dependence) at large Λ , namely $\propto \Lambda^3$. If we take $H(u) \sim 1$ instead, we get a result which behaves like $\ln \Lambda$, a much milder dependence. In any case, we cannot have a too large volume in this case. But again the parametric dependence of the potential correction (3.4) (at least in the $H(u) \sim 1$ case) is $\sim g_s p/N$ with respect to the leading term (3.2).

Hence the impact of spiral may be neglected as before. We should also mention however that the calculation of the impact of the spiral has a potential caveat: we calculate the interaction of the two probes (spiral and sliding) via the modification of the background, but we assume that the only modification when changing a brane to an antibrane is the change in relative sign of the CS and DBI terms, but there is no backreaction on the first probe brane. If we do the interaction in flat space, this approximation would not be valid, however because the calculation is done via the effect on a nontrivial background, it is likely to be valid. In any case, we only wanted to show the correction is small, we will not use the form of (3.4) in the following.

This calculation then looks promising for cosmology, due to the flatness at large r , but we will see in the next section that we have the usual problem that for the cosmology agrees with experiments only for non-generic initial conditions.

4 Cosmology of the model and experimental constraints

Given a potential with a sufficiently flat region, we can use the standard formulas for inflation to determine the constraints on the parameters. An issue we want to examine is the generality of initial conditions. We therefore express the formulas in terms of the fundamental Planck scale m , α' and the 4d Planck scale M_P . The relations of these scales are

$$\begin{aligned} 2\kappa_{10}^2 &= (2\pi)^7 \alpha'^4 = m^{-8} \\ M_P &= m^4 \sqrt{RV_5} \end{aligned} \quad (4.1)$$

where V_5 is the volume of the 5d space, including the metric in front of dx_6^2 , and κ_{10} is the 10d Newton constant. The 4-brane tension is

$$T_4 = \frac{1}{2\sqrt{\pi\alpha'}\kappa_{10}} = \frac{m^4}{\sqrt{2\pi\alpha'}} \quad (4.2)$$

4.1 The model of a sliding brane on a cigar background

The potential before adding the spiral is given by (2.4) plus (2.11), but it is not yet written in terms of the canonical scalar ϕ .

From (2.3), the kinetic term for r is

$$-\frac{T_4 R}{g_s f^{3/2}(r)} \frac{\partial_\mu r \partial^\mu r}{2} \quad (4.3)$$

Since we are interested in the regime $r/r_H \ll 1$, we can put $f(r) \simeq 1$ in the denominator and define the canonical scalar as $\phi = r\sqrt{T_4 R/g_s}$, and for r_H and r_4 corresponding ϕ_H and ϕ_4 . We obtain on the plateau

$$V(\phi) \simeq \frac{T_4 R}{g_s} \left(\frac{\phi_H}{\phi_4} \right)^3 \left[N - \frac{1}{2\alpha_4} \left(1 + \frac{\phi_H^3}{4\phi^3} - \frac{\phi^3}{\alpha_4 \phi_4^3} \right) \right] \quad (4.4)$$

In order to get a good model of inflation, we need that the slow-roll parameters ϵ and η defined by

$$\begin{aligned} \epsilon &\equiv \frac{M_P^2}{2} \left(\frac{V'}{V} \right)^2 \\ \eta &= M_P^2 \frac{V''}{V} \end{aligned} \quad (4.5)$$

are much smaller than 1, since they give the spectral index of primordial scalar fluctuations, $P_\delta^s(k) \sim k^{n_s-1}$,

$$n_s - 1 \equiv \frac{d \ln P_\delta^s(k)}{d \ln k} = -6\epsilon + 2\eta \quad (4.6)$$

which is almost exactly flat ($n_s = 1$). We also need that the number of e-foldings during inflation, given in terms of the potential by

$$\mathcal{N} \equiv \int \frac{H_a}{\dot{\phi}} d\phi = \int_{\phi_{end}}^{\phi_i} \frac{V}{M_P^2 V'} d\phi = \frac{1}{M_P} \int_{\phi_{end}}^{\phi_i} \frac{d\phi}{\sqrt{2\epsilon}} \quad (4.7)$$

(where H_a is the Hubble constant) is larger than about 60, the COBE normalization constraint for the value of the potential at the end of inflation, and finally that we get a sufficiently large reheating temperature T_H at the end of inflation, which generically requires that the potential energy at the start of inflation be not too far below the Planck scale. However, depending on the model, in particular on the couplings to matter, we can have smaller reheating temperature, so we will not use the above for any constraint.

From the approximate potential (4.4) over the flat region $r_H \ll r \ll r_4$, we get

$$\begin{aligned} \epsilon &\equiv \frac{1}{2} \left(M_P \frac{V'}{V} \right)^2 \simeq \frac{1}{2} \left[\frac{3M_P}{2N\phi} \left(\frac{1}{4} \frac{\phi_H^3}{\phi^3} + \frac{\phi^3}{\phi_4^3} \right) \right]^2 \\ \eta &\equiv M_P^2 \frac{V''}{V} = \frac{3M_P^2}{2N\phi^2} \left(-\frac{\phi_H^3}{\phi^3} + 2\frac{\phi^3}{\phi_4^3} \right) \end{aligned} \quad (4.8)$$

which can be very small, even if M_P/ϕ is generic or even large (small ϕ). All we needed for this result was $\phi_H \ll \phi \ll \phi_4$, which is a generic case, since we saw that we needed ϕ_H to be small for our construction to be valid (one could argue whether such a ϕ_H is natural or not) and ϕ_4/M_P is generically large, as seen from (4.16) (since N is large and g_s is small, we would need large V_5 to obtain small ϕ_4/M_P , which is possible, but non generic). Therefore a generic ϕ , of order M_P , will fall within the plateau regime.

Experimentally, a red spectrum ($n_s - 1 < 0$) is preferred (see for instance [11]). Since from the above we have generically $\epsilon \ll |\eta|$, the condition for a red spectrum is $\eta < 0$, or $\phi < \sqrt{2^{-1/3} \phi_H \phi_4}$.

For the number of e-folds we obtain

$$\mathcal{N} \simeq \frac{2N}{3} \int_{\phi_{end}}^{\phi_{in}} \frac{d\phi}{M_P} \frac{\phi/M_P}{\frac{\phi_H^3}{4\phi^3} + \frac{\phi^3}{\phi_4^3}} \quad (4.9)$$

Assuming we can neglect ϕ^3/ϕ_4^3 with respect to the first term over the period of interest, we finally obtain

$$\mathcal{N} = \frac{8N}{15} \frac{\phi_{in}^5 - \phi_{end}^5}{M_P^2 \phi_H^3} \quad (4.10)$$

which can easily be made large enough.

The COBE normalization constraint states that the magnitude of the scalar fluctuations during inflation (specifically, at horizon exit, but since we are on the plateau, what is constrained is the plateau value),

$$\langle \Delta\phi \rangle = \frac{H_e}{2\pi} \quad (4.11)$$

must give rise to the observed CMBR fluctuations, i.e. must equal $\sqrt{2\epsilon}10^{-5}M_P$ ($\delta\rho/\rho = H_e/(2\pi\sqrt{2\epsilon}M_P)$). This puts a constraint on the magnitude of the potential on the inflation plateau V_p , since $2\pi V_p/3 = H_e^2 M_P^2$, giving

$$V_p \sim 12\pi\epsilon \times 10^{-10}M_P^4 \quad (4.12)$$

Since we have

$$V_p = N \left(\frac{T_4 R}{g_s} \right) \left(\frac{\phi_H}{\phi_4} \right)^3 \quad (4.13)$$

we can think of this as a constraint on $(T_4 R/g_s)$ once N and ϕ_H^3/ϕ_4^3 are fixed by (4.8) and (4.10).

Reheating and relaxing to zero potential

At $r = r_H$ we have $V'(r \sim r_H) \sim 1/\sqrt{f} \rightarrow \infty$, as we saw. It is true that then the kinetic term for r must be put in the canonical form, however from (2.3) we see that at $r \simeq r_H$ we don't have a nonlinear sigma model, but rather we have a nonstandard and divergent kinetic term $\sim \sqrt{(\partial\phi)^2/(r - r_H)}$, signifying that the effective description in terms of a single scalar field is breaking down. Before that however, the derivative of the scalar potential will become large, and inflation will end, so the breakdown region corresponds to the where reheating should take place.

Therefore in this region string corrections should become important, at least in the interesting case when the number of anti-branes making the background nonextremal, δN , is of order 1. In this case we know that there is not much reason to consider δN as part of the background, while one brane is kept as a probe, at least not when the probe is close to the antibranes (located near $r = 0$, or $r = r_H$). Instead a better description would involve antibranes at $r = 0$, which would annihilate with the sliding brane, generating string corrections. So the slope of the potential at r_H , as well as its depth at r_H , could be corrected anyway. In any case, the nonsusy background will be unstable, so some time after the probe brane hits r_H we should have a decay process, made favorable also by this collision, and we should decay to a supersymmetric vacuum of zero potential.

This process will reheat the Universe, through decay into matter modes of the energy released in the fall to the susy vacuum, as usual in brane-antibrane inflation,

see for instance [3]. In principle, two possible scenarios can occur: the potential could have zero slope at $r = r_H$, and we could have standard reheating through oscillations. Or, in view of the above, more likely is the usual brane-antibrane case, of a very steep potential, giving rise to preheating (see [12] for a review). But as usual, this is a complicated nonperturbative process. If the potential is steep enough and the coupling to matter large enough, we expect to generate a large enough reheating temperature, though we will not attempt a further description.

In conclusion, in this model we can easily satisfy experimental constraints with generic initial conditions for ϕ .

Of course, the usual caveat present in [2] still applies in the same form, since the supersymmetric case is T dual to the one considered there. When we try to stabilize the volume modulus, generically we will spoil the slow-roll conditions of the potential.

4.2 The model of a sliding anti-brane on a cylinder background

In this case, $f = 1$, so the kinetic term has only a constant rescaling, i.e. $\phi = r\sqrt{T_4 R/g_s}$, giving

$$V_4(\phi) = \frac{2T_4 R}{g_s} \frac{1}{1 + \frac{\phi_4^3}{\phi^3}} \quad (4.14)$$

The slow roll parameters ϵ, η are

$$\begin{aligned} \epsilon &= \frac{1}{2} \left[\frac{3M_P}{\phi} \frac{\phi_4^3/\phi^3}{1 + \phi_4^3/\phi^3} \right]^2 \\ \eta &= -\frac{M_P^2 \phi_4^3/\phi^3 (32 + 30\phi_4^3/\phi^3)}{\phi^2 (1 + \phi_4^3/\phi^3)^2} \end{aligned} \quad (4.15)$$

But ϕ_4 in Planck units is

$$\frac{\phi_4}{M_P} = \left[\frac{\pi^{1/4}}{2^{3/4} \sqrt{g_s}} \right]^{1/3} \sqrt{\frac{\sqrt{\alpha'} N^{2/3}}{m^4 V_5}} \quad (4.16)$$

We then see that in order to have a small ϵ and η we can either have $\phi \gg M_P$ which is non-generic (and difficult to obtain, as type IIB examples show) or, if instead $\phi \sim M_P$, we need $\phi \gg \phi_4$, which means we need $\phi_4/M_P \ll 1$. We see that this latter case is only possible if $V_5 m^4 / (\sqrt{\alpha'} N^{2/3}) \gg 1$, which is possible, but is again non-generic. If we allow for either of these two non-generic cases however, we can also get a large enough number of e-folds, since

$$\mathcal{N} = \int_{\phi_{end}}^{\phi_{in}} d\phi \frac{\phi^4 (1 + \phi_4^3/\phi^3)}{3M_P^2 \phi_4^3} \simeq \frac{\phi_{in}^5 - \phi_{end}^5}{3M_P^2 \phi_4^3} \quad (4.17)$$

where in the second equality we have assumed $M_P \sim \phi \gg \phi_4$. The COBE normalization gives now

$$V_f \sim \frac{2T_4 R}{g_s} \sim 12\pi\epsilon \times 10^{-10} M_P^4 \quad (4.18)$$

We also note now that, since $\epsilon > 0$ by definition and $\eta < 0$ in this case, we always have the experimentally preferred red spectrum.

Reheating is simpler in this model, since now we already know that inflation ends at $\phi = 0$, where $V(0) = 0$, where all the energy goes into matter modes, and moreover

$$\frac{dV_4}{d\phi}(\phi) = \frac{6T_4 R}{g_s \phi} \frac{\phi_4^3/\phi^3}{1 + \phi_4^3/\phi^3} \quad (4.19)$$

so the slope of the potential in canonical scalar variable blows up at $\phi = 0$, and we have the usual preheating scenario. Note that in this case, due to the supersymmetry of the background, it is hard to see how there could be string corrections to the potential near $r = 0$, so the fact that the derivative of the potential blows up seems robust.

5 Conclusions

In this paper we have analyzed type II_A inflation scenarios based on the MQCD model [1]. The supersymmetric case, T dual to a compactified KS model, is a Wick-rotated D4-brane with a spiral 5-brane added, and compactified by gluing a CY space. For a model of inflation we considered two cases.

For a near-extremal background with a moving D4-brane probe, we obtained a flat enough potential, with slow-roll conditions and normalization which are easy to satisfy for generic initial conditions. We also saw that the interaction of the probe brane with the spiral is negligible in the final potential. Reheating in this model is harder than usual to analyze because the small r region is unreliable in this probe brane approximation, especially if $\delta N = 1$ in the background. We also argued that the additional energy due to the gluing of the CY, and modifications of the near-extremal background due to the probe brane in other quantities than H are small, but it would be useful to find a way to calculate them directly.

In the second model, we considered a supersymmetric background, with an anti-D4 probe brane. Then the gluing of the CY doesn't introduce extra energy, but there are several potential caveats for interaction of the probe with the spiral. We have seen however that likely we can again ignore this correction to the potential. The model can produce slow-roll inflation, and reheating is easier to describe, but the initial conditions for inflation are non-generic.

We note that the final inflationary models obtained were not simply related to the KKLMNT one. The supersymmetric version of our models was related by T-duality, but the supersymmetry breaking was enough to guarantee new results for cosmology. It will be interesting to study further the physical implications of these models.

In both of these models, we have the usual problem of that the stabilization of the volume modulus generically spoils the slow-roll, in a similar manner to the KKLMNT model. Since [2], a lot of work has been done on the problem of moduli stabilization in the context of brane sliding on a throat stabilized by fluxes, for example [13, 14, 15, 16, 17, 18, 19, 20]. For some recent reviews of string inflation with a more complete list of references, see [21, 22, 23]. Generically, the stabilization of the moduli gives a large mass to the inflaton (brane position), spoiling slow-roll, but there are extra contributions which in principle could cancel this (see e.g. [16]) in a fine-tuned manner. Otherwise, one would have to rely on fine-tuned initial conditions for inflation. The problem is still that in order for this to work, one would have to consider a comprehensive modular potential, including all possible contributions, and that is a very difficult task. Progress in this direction was realized in [17, 19]. In a general potential, there are too many contributions, but in [20], a statistical approach revealed that in a small (fine-tuned) set of potential coefficients can give rise to inflation, independent on initial conditions. However, here we have not attempted to address the issue of moduli stabilization, since as described above it is a complicated issue and we leave it to future investigation. Having saying that, let us speculate about a possible scenario for the stabilization of the volume modulus. In analogy with the mechanism in the KKLT model, one can contemplate adding a D8 brane that wraps the circle as well as the S^4 . Such a brane may yield a gaugino condensation and thus produce a volume-stabilizing potential. The addition of such a brane should not spoil the slow-roll properties of the inflaton potential since it will be suppressed by powers of $1/N$ in a similar manner to the suppression of the contribution to the potential from the spiral brane discussed above.

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